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Branes at singularities in Type 0 string theory

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ABSTRACT: We consider Type 0B D3-branes placed at conical singularities and analyze in detail the conifold singularity. We study the non supersymmetric gauge theories on their worldvolume and their conjectured dual gravity descriptions. In the ultraviolet the solutions exhibit a logarithmic running of the gauge coupling. In the infrared we find confining solutions and IR fixed points.

KEYWORDS: Confinement, D-branes, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

The Type 0 string theories have worldsheet supersymmetry but no space-time supersymmetry as a consequence of non-chiral GSO projection [1, 2]. Consider two types of such string theories, Type 0A and Type 0B. They do not have space-time fermions in their spectra. Nevertheless, they have a modular invariant partition function. The bosonic fields of these theories are as those of the supersymmetric Type IIA and Type IIB string theories, with a doubled set of Ramond-Ramond fields. In addition they contain a tachyon field T.

Type 0 theories have D-branes. As in the Type II case, one can consider the gauge theories on the worldvolume of N such branes. These theories do not contain an open string tachyon [3]. Moreover, the bulk tachyon can condense due to its coupling to the Ramond-Ramond fields.

One example studied in [4, 5, 6, 7] is the theory on D3 branes in Type 0B theory. Since there is a doubled set of RR 4-form fields in Type 0B string theory, the D3 branes can carry two charges, electric and magnetic. The worldvolume theory of N flat electric D3 branes is a $\mathrm{U}(N)$ gauge theory with six scalars in the adjoint representation of the gauge group.

Asymptotic solutions of a proposed dual gravity background were constructed in [4]. At large radial coordinate the tachyon is constant and one finds a metric of the form $AdS_5 \times S^5$ with vanishing coupling which was interpreted as a UV fixed point. The solution exhibits a logarithmic running in qualitative agreement with the asymptotic freedom property of the field theory [5, 6]. Moreover the sign of the second coefficient of the β-function was found to be in agreement with field theory [5].

At small radial coordinate the tachyon vanishes and one finds again a solution of the form $AdS_5 \times S^5$ with infinite coupling, which was interpreted as a strong coupling IR fixed point [5]. The complete gravity solution describing the flow from the UV fixed point to the IR fixed point has not been constructed yet.

Generically one expects the gauge theory to have different phases parametrized by the possible couplings. The IR fixed point should occur as a particular tuning of the couplings. Indeed, other solutions at small radial coordinate were constructed in [7] that exhibit confinement and mass gap. Moreover they were argued to be more generic than the IR fixed point solution.

One can construct other non supersymmetric theories by placing the Type 0B D3 branes at singularities. In this paper we will pursue this direction. We will mostly consider the theories obtained by placing Type 0B D3 branes at a conifold singularity, but we will also discuss general features applicable to other singularities as well. The paper is organized as follows. In the next section we will comment on the phase structure of the worldvolume theory of N flat electric D3 branes of Type 0B string theory and the dual gravity description. The discussion will apply as well to branes at singularities which will be studied later. In section 3 we will place Type 0B D3 branes at a conifold singularity. We will consider electric branes, magnetic branes and self-dual branes. We will discuss the worldvolume field theory and construct conjectured dual gravity descriptions. These exhibit UV freedom and a logarithmic running of the gauge coupling. However, we will see that the sign of the two-loop β-function coefficient is not captured correctly by gravity. In the IR we find gravity solutions that correspond to fixed points as well as solutions that exhibit confinement. In section 4 we will discuss Type 0B D3 branes placed in general singular spaces. Section 5 is devoted to a discussion on the RG flow from the UV to the IR in the gravity description.

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2. Comments on phase structure

In the following we make some comments that will be relevant later too, on the theory of N flat electric D3 branes of Type 0B string theory. The theory is a non supersymmetric SU(N) gauge theory with six real scalars $X^i, i = 1, \ldots, 6$ in the adjoint representation of the gauge group. The classical action is derived by a dimensional reduction of the pure SU(N) gauge theory action in ten dimensions. The six scalars are the components of the gauge fields in the reduced dimensions.

The theory has classically SO(6) global symmetry that rotates the six scalars. This allows three possible parameters: a gauge coupling g_{YM}, a mass parameter for the scalars m and a scalar quartic potential coupling g. In the classical action, the mass parameter is zero and g is fixed in terms of g_{YM}. Quantum mechanically, the three parameters are corrected differently and can take independent values. The theory has a phase diagram depending on these three parameters. Generically we expect to see in the diagram Coulomb-like (Higgs) phases, confinement phases and maybe non trivial RG fixed points arising from particular tuning of the parameters. For instance, such a picture has been seen on the lattice for SU(2) gauge theory with one scalar in the adjoint representation [8, 9, 10]. One starts with the theory with three bare parameters, the gauge coupling, a mass parameter for the scalar and a scalar quartic potential coupling. The phase diagram is constructed by tuning the bare parameters to different values and taking the continuum limit. The case with six scalars in the adjoint representation has not been analysed on the lattice yet.

We may expect, like in the case with one scalar, that the vacuum expectation value for the scalars will vanish in the confining phase and be non zero in the Higgs phase. A non trivial fixed point may appear on the border of these phases at zero vacuum expectation value for the scalars. Since the gravity description that we consider corresponds to D3 branes on top of each other, the vacuum expectation value for the scalars is zero. We expect to see then a confining phase and maybe IR fixed points.

The electric D3 branes theory is conjectured to have a dual string description. There are two approximations of the dual description that can be taken. When the effective string coupling e^{ϕ} is small string loops are suppressed and we can use the classical string description. When the α' corrections are small the classical gravity description is applicable.

The gauge theory is asymptotically free. At short distances the theory is weakly coupled and we expect the dual gravity description not to be valid. This can happen for instance if the gravity background is singular as in the models based on heating up theories in one higher dimension [11, 12]. In the case at hand the signal for the gravity description being non adequate is that the α' corrections are of the same order as the leading order gravity contributions [5]. For large N the curvature in string units is small but α' corrections associated with the Weyl tensor are not suppressed. The effective string coupling is small and therefor we expect the classical string description to be applicable. This still has an advantage over the finite temperature case in that we can see qualitatively the asymptotic freedom property already in the leading order gravity description. In [5] it was argued that the gravity description at large radial coordinate captures the right sign of the one and two loop β function coefficients.

The gravity description cannot capture quantitatively the renormalization group flow from the UV to the IR. For the electric D3 branes there is a gravity solution that indicates an IR fixed point at infinite coupling. While the α' corrections are suppressed in this region, the dilaton is large and classical string theory is not sufficient to study the fixed point theory. The gravity solution at all energy scales u has not been constructed yet. Other gravity solutions with a confining behaviour in the IR were found in [7] and will be discussed in the following.

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The ansatz for the background metric (in string frame) is [4]

$$ds = e^{\phi/2}\left(e^{\frac{1}{2}\xi-5\eta}d\rho^2 + e^{-\xi/2}dx^\mu dx^\mu + e^{\frac{1}{2}\xi-\eta}d\Omega_5^2\right), \tag{2.1}$$

where ϕ, ξ, η are functions of the radial coordinate $\rho \sim 1/u^4$ and ϕ denotes the dilaton field. The tachyon field T induces a change in the dilaton field in the radial coordinate ρ. This corresponds to the running of the gauge coupling in the gauge theory. Different forms of the tachyon fields as a function of ρ induce different RG trajectories from the UV to the IR. Changing the form of the tachyon field and as a consequence of the equations of motion changing the form of ϕ, ξ, η is analogus to changing the parameters g_{YM}, m, g. In analogy with the lattice picture, we expect the gravity description to exhibit different phases parametrized by the form of ϕ, ξ, η and T.

The mapping of parameters between the field theory and gravity cannot be made quantitatively precise. One can see, however, that ξ, η together with ϕ control the size of the S^5 part of the metric (2.1). One expects that the signal for the scalars in the adjoint representation being massive is the shrinking of the five sphere to zero size.

Consider the following large ρ behaviour of ϕ, ξ, η [7]

$$\begin{aligned}\phi &\sim \alpha_0\rho + \phi_0 \\ \xi &\sim \alpha_1\rho + \xi_0 \\ \eta &\sim \alpha_2\rho + \eta_0\,,\end{aligned} \tag{2.2}$$

with a non zero tachyon and the constraints

$$\frac{1}{2}\alpha_0^2 + \frac{1}{2}\alpha_1^2 - 5\alpha_2^2 = 0\,, \tag{2.3}$$

and $\alpha_i > 0, i = 0, 1, 2$. With the parametrization (2.2) the metric (2.1) takes the form

$$ds = e^{(\frac{1}{2}\alpha_0+\frac{1}{2}\alpha_1-5\alpha_2)\rho}d\rho^2 + e^{(\frac{1}{2}\alpha_0-\frac{1}{2}\alpha_1)\rho}dx^\mu dx^\mu + e^{(\frac{1}{2}\alpha_0+\frac{1}{2}\alpha_1-\alpha_2)\rho}d\Omega_5^2\,. \tag{2.4}$$

The large $\rho \sim 1/u^4$ region corresponds to the IR region of the gauge theory. To analyse the phase of the corresponding gauge theory as a function of the gravity parameters α_i we can use the Wilson loop as an order parameter. Its computation is done as in [13, 14], by minimization of the action of a string with worldsheet bounded by the loop. The theory is in a confining phase if the $g_{\mu\mu}$ component of the metric (2.4) does not vanish anywhere. In particular it must not vanish as $\rho \to \infty$. The reason is that the string tends to minimize its length by going to the region with the smallest possible value of $g_{\mu\mu}$, it will stay there for most of the geodesic and then will go up to the position of the other external charge [15, 16]. Therefore the energy is proportional to the distance of the charges and the string tension is $\mathrm{Min}(g_{\mu\mu}(\rho))$ up to a numerical constant. On the other hand if there is a zero at the horizon there are two possibilities: either the quark antiquark potential is Coulombic [13, 14] or the energy is minimized by two straight strings which can be separated without cost of energy which corresponds to electric screening. In [7] it was observed that the second possibility is realized for backgrounds of the form (2.2). Thus, the background (2.4) corresponds to a phase with electric screening for $\alpha_0 < \alpha_1$ and to a confining phase for $\alpha_0 \geq \alpha_1$. Using calculations along the lines of [12, 7] one can show the existence of a mass gap in the confining phase. The radius of the five-sphere is controlled by $\alpha_0 + \alpha_1 - 2\alpha_2$. When this quantity is negative the sphere shrinks to zero size in the IR which signals that the scalars are massive and that the theory may be in the same universality class as SU(N) Yang-Mills theory. These solutions at large ρ have not been connected to the UV region at small ρ yet. With a vanishing tachyon at large ρ, ξ and η approach a constant value and there is a large ρ solution of the form $AdS_5 \times S^5$ that signals the IR fixed point discussed above [5].

An important issue in the gravity description is the tachyon instability [4]. A possible mechanism for removing it employs the RR fields of Type 0 theories, since the effective action contains the terms

$$S_{eff} \sim \int \sqrt{G}\left(m^2 e^{-2\Phi} T^2 + F^2 T^2\right), \tag{2.5}$$

where F is the RR five form field strength. The coupling of T and F in (2.5) can shift the effective tachyon mass to be positive. Assuming the overall scale of the metric to be $(g_s N)^{1/2}$ results in a stability condition $g_s^2 l_s^2 |F|^2 > O(1)$. Since $F \sim N$ we get $g_s N < O(1)$. Therefor the stability condition appears to hold for small bare 't Hooft coupling which is the correct limit to take in order to get to the continuum theory. The same conclusion applies to the cases where the branes are placed at singularities which we will study next.

3. D3 branes in conifold background

In this section we will study the theory obtained by placing Type 0B D3 branes at a conifold $\mathcal{C}$.

3.1 Field theory

When placing N D3 branes of Type IIB string theory at a conifold it is argued [17] that the resulting worldvolume theory is $\mathcal{N} = 1$ supersymmetric SU(N) $\times$ SU(N) gauge theory with chiral superfields $A_k, k = 1, 2$ transforming in the $(N, \bar{N})$ representation and $B_l, l = 1, 2$ transforming in the $(\bar{N}, N)$ representation. In the IR the theory flows to a non trivial fixed point with the superpotential $W = \frac{t}{2}\varepsilon^{ij}\varepsilon^{kl} Tr A_i B_k A_j B_l$ becoming exactly marginal.

Electric or magnetic D3 branes On the worldvolume of N electric D3 branes of Type 0B string theory at a conifold we expect a truncation of the fermions, namely, an $\mathrm{SU}(N)\times\mathrm{SU}(N)$ gauge theory with complex scalar fields $A_k, k=1,2$ transforming in the $(N,\bar{N})$ representation and $B_l, l=1,2$ transforming in the $(\bar{N},N)$ representation. Classically the theory has $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ global symmetry which can be viewed as inherited from the $\mathcal{N}=1$ supersymmetric theory upon truncating the fermionic fields. One SU(2) acts on A_k and the other SU(2) acts on B_l. The U(1) charges of A_k and B_l are opposite.

With this global symmetry there are several possible bare parameters. Let us assume the the matrices A_k, B_l are diagonal in some basis. There are two gauge couplings g_{YM_1}, g_{YM_2}, two possible mass terms $m_1^2 Tr A_k\bar{A}_k, m_2^2 Tr B_l\bar{B}_l$ and several possible quartic couplings $g_1 Tr(A_k\bar{A}_k)^2$, $g_2 Tr(A_k\bar{A}_k)(B_l\bar{B}_l)$, $g_3 Tr(B_l\bar{B}_l)^2$ and squares of the mass terms. However, the eigenvalues a_k, b_l of the matrices A_k, B_l respectively parametrize the positions of the N D3 branes at points in conifold and are related by the equation defining the conifold, $a_k\bar{a}_k = b_l\bar{b}_l$ modulo the U(1) action. This leaves us with one mass parameter m and one quartic coupling g. By symmetry argument one can identify the bare gauge couplings parameters which we will call g_{YM}.

The Yang-Mills coupling at energy scale μ

$$g_{YM}^2(\mu) = \frac{8\pi^2}{b_1 \log\frac{\mu}{\mu_0} + \frac{b_2}{2b_1^2}\log\log\frac{\mu}{\mu_0}}\,, \tag{3.1}$$

where b_1 and b_2 are the one-loop and two-loop beta functions coefficients. The one-loop β-function coefficient is $b_1 = 3N$ and the theory is asymptotically free. With the two equal gauge couplings the two-loop β-function coefficient at large N is $b_2 = 2N^2$. Therefor, unlike the theory of N electric D3 branes in flat space, here $\frac{b_2}{2b_1^2} = \frac{1}{9} > 0$. As we will see later, while the gravity description captures the asymptotic freedom property of this theory it does not capture correctly the sign of the two-loop β-function. The theory has a phase diagram depending on the three parameters g_{YM}, m, g. We expect to see in the diagram Coulomb-like phases, confinement phases and maybe non trivial RG fixed points arising from particular tuning of the parameters.

The worldvolume theory of N magnetic D3 branes of Type 0B string theory is the same as that of N electric ones.

Self-dual D3 branes The theory of N self-dual D3 branes consists of N electric and N magnetic D3 branes. The worldvolume theory of N self-dual D3 branes at a conifold is a gauge theory with gauge group $\mathrm{SU}(N)^4$. The matter content consists of $8N^2$ complex scalars and $12N^2$ Weyl fermions. Their representation can be deduced as in [18]. The $8N^2$ complex scalars transform in the bi-fundamental representations

$$2\Big((N,\bar{N},1,1)\oplus(\bar{N},N,1,1)\oplus(1,1,N,\bar{N})\oplus(1,1,\bar{N},N)\Big)\,, \tag{3.2}$$

and the Weyl fermions in

$$2(N,1,\bar{N},1)\oplus 2(\bar{N},1,N,1)\oplus 2(1,N,1,\bar{N})\oplus 2(1,\bar{N},1,N)\oplus \\ \oplus(N,1,1,\bar{N})\oplus(\bar{N},1,1,N)\oplus(1,N,\bar{N},1)\oplus(1,\bar{N},N,1)\,. \quad (3.3)$$

The fermions arise from the open strings stretched between the electric and the magnetic branes [19].

The worldvolume theory is asymptotically free. We will see that the gravity picture indicates that for zero vacuum expectation value for the scalars the theory flows in the IR to a fixed point.

3.2 Type 0B gravity description

In this section we will paramatrize the ten dimensional metric in the Einstein frame by

$$ds_E^2 = e^{\frac{1}{2}\xi-3\lambda+\eta}d\rho^2 + e^{-\frac{1}{2}\xi}dx^\mu dx^\mu + e^{\frac{1}{2}\xi+\lambda-3\eta}(d\psi+\cos(\theta_1)d\phi_1+\cos(\theta_2)d\phi_2)^2 + \\ +e^{\frac{1}{2}\xi-\lambda+\eta}\sum_{i=1,2}(d\theta_i^2+\sin(\theta_i)^2 d\phi_i^2)\,, \quad (3.4)$$

with the string metric given by $ds^2 = e^{\frac{1}{2}\phi}ds_E^2$.

In this parametrization the effective one dimensional action for radial evolution of the fields takes the form

$$S=\int d\rho\left[\frac{1}{2}\phi'^2+\frac{1}{2}\xi'^2-\lambda'^2+3\eta'^2-2\lambda'\eta'+\frac{1}{4}T'^2-V(\phi,\xi,\lambda,\eta,T)\right]. \quad (3.5)$$

The potential V is given by

$$V = g(T)e^{\frac{1}{2}\phi+\frac{1}{2}\xi-3\lambda+\eta} - e^{-4\eta} + 4e^{-2\lambda} - h(T)e^{-2\xi}\,, \quad (3.6)$$

where

$$h(T) = P^2 f(T) + \frac{Q^2}{f(T)}\,, \quad (3.7)$$

with $f(T)=1+T+T^2/2$, and we set the string scale l_s to one. The function $h(T)$ arises from the coupling of the tachyon to electric $Q\neq 0$ and magnetic $P\neq 0$ D3 branes. Furthermore, the equations of motion of (3.5) are supplemented by a zero energy constraint

$$\frac{1}{2}\phi'^2+\frac{1}{2}\xi'^2-\lambda'^2+3\eta'^2-2\lambda'\eta'+\frac{1}{4}T'^2+V(\phi,\xi,\lambda,\eta,T)=0\,. \quad (3.8)$$

We will consider a quartic potential $g(T)$ for the tachyon field in (3.6)

$$g(T) = \frac{1}{2}T^2 - tT^4\,. \quad (3.9)$$

3.2.1 Electric D3 branes

In the following we will study the theory of electric threebranes ($P = 0$) at a conifold singularity. We will start by setting $g(T) = \frac{1}{2}T^2$ which is the usual mass term for the tachyon. We will include the quartic coupling (3.9) later. The equations of motion following from (3.5) take the form

$$
\begin{aligned}
\phi'' + \frac{1}{4}T^2 e^{\frac{1}{2}\phi+\frac{1}{2}\xi-3\lambda+\eta} &= 0\,, \\
\xi'' + \frac{1}{4}T^2 e^{\frac{1}{2}\phi+\frac{1}{2}\xi-3\lambda+\eta} + 2Q^2 f^{-1} e^{-2\xi} &= 0\,, \\
\lambda'' + \eta'' + \frac{3}{4}T^2 e^{\frac{1}{2}\phi+\frac{1}{2}\xi-3\lambda+\eta} + 4e^{-2\lambda} &= 0\,, \\
-2\lambda'' + 6\eta'' + \frac{1}{2}T^2 e^{\frac{1}{2}\phi+\frac{1}{2}\xi-3\lambda+\eta} + 4e^{-4\eta} &= 0\,, \\
T'' + 2T e^{\frac{1}{2}\phi+\frac{1}{2}\xi-3\lambda+\eta} + 2Q^2 \frac{f'}{f^2} e^{-2\xi} &= 0\,.
\end{aligned}
\tag{3.10}
$$

UV freedom In the UV we assume that the tachyon condensenses $T = -1$ and in order to find a solution we first solve the equations for massless tachyon and consider the mass term as a small perturbation around the zeroth order equations. The zeroth order solution is just $AdS_5 \times S^5$ with constant dilaton and takes the form

$$
\begin{aligned}
T = -1\,, \qquad \phi = \phi_0\,, \qquad \xi = \log(2Q) - y\,, \\
\eta = \frac{1}{4}\log(4) - \frac{y}{2}\,, \qquad \lambda = \frac{1}{2}\log(8/3) - y\,,
\end{aligned}
\tag{3.11}
$$

where

$$
\rho = e^{-y}\,. \tag{3.12}
$$

ρ is related to the radial variable u by $\rho = u^{-4}$. In the UV region $y \gg 1$ and we find an asymptotic solution to the equations (3.10)

$$
\begin{aligned}
T &= -1 + \frac{8}{y} + \frac{156}{y^2}\log(y) + \cdots \\
\phi &= \log\left(\frac{2^{13}}{27Q}\right) - 2\log(y) + \frac{39}{y}\log(y) + \cdots \\
\xi &= \log(2Q) - y + \frac{1}{y} + \frac{39}{2y^2}\log(y) + \cdots \\
\eta &= \frac{1}{4}\log(4) - \frac{y}{2} + \frac{1}{y} + \frac{39}{2y^2}\log(y) + \cdots \\
\lambda &= \frac{1}{2}\log\left(\frac{8}{3}\right) - y + \frac{2}{y} + \frac{39}{y^2}\log(y) + \cdots
\end{aligned}
\tag{3.13}
$$

The Einstein metric is asymptotic to $AdS_5 \times T^{1,1}$ where

$$ds^2_{T^{1,1}} = \left[\frac{1}{9}\Big(d\psi + \cos(\theta_1)d\phi_1 + \cos(\theta_2)d\phi_2\Big)^2 + \frac{1}{6}\sum_{i=1,2}\Big(d\theta_i^2 + \sin(\theta_i)^2 d\phi_i^2\Big)\right], \quad (3.14)$$

which indicates a UV fixed point. The effective string coupling vanishes at infinite u and the corresponding gauge theory has a vanishing coupling in accord with the UV freedom of the theory.

In fact, it follows from the equations (3.10) and can be seen from the solutions (3.13) that we can identify $\lambda = 2\eta$ up to a constant term. With such an identity the whole metric can be rewritten as a five dimensional metric times the metric of the $T^{1,1}$ manifold multiplied by a ρ dependent function. As we will discuss in the next section this is an example of a general feature, namely the effective gravity solution has the form of a five dimensional space whose form is universal times a five dimensional Einstein space (with a warp factor). The relevant equations of ϕ, ξ and η are universal and do not depend on the background in which we put our D3 branes.

For D3 branes in flat space the numerical values of the two loop beta function in field theory and gravity do not agree, but the sign matches. The sign difference the b_1 and b_2 coefficients was taken as an evidence that the field theory may run to an IR fixed point.

As in [5], from the form of the dilaton we can read off the running of the gauge coupling

$$Q^{1/2}e^{\frac{1}{2}\phi} \sim \frac{1}{\log u - \frac{39}{8}\log\log u}, \quad (3.15)$$

which exhibits a logarithmic running. However comparing (3.15) to (3.1) we see that in the gravity description $b_2/2b_1^2 = -39/8 < 0$, while the field theory predicts $b_2/2b_1^2 = 1/9 > 0$. Thus, for the conifold model neither the numerical values agrees nor the sign. This, as we will see later, is not special to the conifold model and generically one cannot trust the sigh of the two-loop β function predicted by gravity.

The sign of b_2/b_1^2 is neither necessary nor sufficient to prove the existence of an IR fixed point in the field theory. As we will see, the gravity description indicates the existence of a such fixed point at infinite coupling.

For $u \gg 1$ the effective string coupling $e^{\phi} \sim 1/Q\log^2 u$. Therefore the string loops are suppressed and the classical string description is applicable. For large Q the curvature in string units is small too. However, the α' corrections associated with the Weyl tensor are of the same order as the leading gravity contribution. Therefor the classical gravity description is not applicable. Indeed it does not lead to a quantitatively correct running of the gauge coupling.

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IR fixed point In the region $\rho = e^y \gg 1$ corresponding to the IR region of the gauge theory we have an asymptotic solution to the equations (3.10) of the form

$$\begin{aligned}
T &= -\frac{16}{y} + \frac{1}{y^2}(-72\log(y) + \cdots) \\
\phi &= \log\left(2^{-5/2}27^{-1}Q^{-1}\right) + 2\log(y) + \frac{1}{y}(-9\log(y) + \cdots) \\
\xi &= \frac{1}{2}\log(2Q^2) + y + \frac{9}{y} + \frac{1}{y^2}\left(\frac{81}{2}\log(y) + \cdots\right) \\
\eta &= \frac{1}{2}\log(2) + \frac{y}{2} + \frac{1}{y} + \frac{1}{y^2}\left(\frac{9}{2}\log(y) + \cdots\right) \\
\lambda &= \frac{1}{2}\log\left(\frac{8}{3}\right) + y + \frac{2}{y} + \frac{1}{y^2}(9\log(y) + \cdots)
\end{aligned} \tag{3.16}$$

In the IR limit $u \to 0$ the tachyon vanishes and the metric in the Einstein frame describes $AdS_5 \times T^{1,1}$. The effective string coupling is infinite in this limit. Therefore the solution indicates an IR fixed point at infinite gauge coupling. The classical string description is not sufficient in order to describe the fixed point. However, away from the fixed point it can provide a good description since $e^\phi \sim 1/Q$ which is small for large number of branes. In the IR the curvature in string units is small and the α' corrections are suppressed. We expect therefor that classical string theory can provide a good description of the whole flow from the UV to the IR besides the fixed point itself

In the gauge theory we expect that a fine tuning of the bare parameters is needed to get to the fixed point. In the supergravity this fine tuning corresponds to a fine tuning of the constants of integration of the IR solution to match the UV solution. In particular we do not expect this fine tuning to be the generic solution.

Confinement To find solutions to (3.10) in the IR that exhibit confinement we will consider also the quartic part of the tachyon field potential (3.9). Consider the case when the tachyon condenses at a non zero value T_0 and the tachyon potential (3.9) obeys the constraint $g'(T_0) = 0$. This implies the tachyon is exactly constant and does not run. For example one could take $f(T) = 1 + T + T^2/2$ and $g(T) = T^2/2 - T^4/4$ with $T_0 = -1$ but the precise form of the tachyon potential is not important.

In this case an asymptotic solution for large ρ is given, up to exponentially suppressed corrections, by

$$\phi \sim \phi_0 + \alpha_0\rho\,, \qquad \xi \sim \xi_0 + \alpha_1\rho\,, \qquad \eta \sim \eta_0 + \alpha_2\rho\,. \tag{3.17}$$

The zero energy condition (3.8) imposes the constraint

$$\frac{1}{2}(\alpha_0^2 + \alpha_1^2) - 5\alpha_2^2 = 0\,. \tag{3.18}$$

The conditions $\alpha_i \geq 0, i = 0, 1, 2$ together with (3.18) guarantee that the corrections from exponential term in the equations of motion (3.10) are suppressed.

With the parametrization (3.17) the Einstein frame metric (3.4), written in the string frame, takes the form

$$ds^2 = e^{(\frac{1}{2}\alpha_0+\frac{1}{2}\alpha_1-5\alpha_2)\rho}d\rho^2 + e^{(\frac{1}{2}\alpha_0-\frac{1}{2}\alpha_1)\rho}dx^\mu dx^\mu + e^{(\frac{1}{2}\alpha_0+\frac{1}{2}\alpha_1-\alpha_2)\rho}ds^2_{T^{1,1}}\,. \qquad (3.19)$$

To analyze the phase of the corresponding gauge theory as a function of the gravity parameters α_i we can use the Wilson loop as an order parameter. The discussion is similar to that in section 2. The background (3.19) corresponds to electric screening for $\alpha_0 < \alpha_1$ and to a confining phase for $\alpha_0 \geq \alpha_1$. The radius of the $T^{1,1}$ is controlled by $\alpha_0+\alpha_1-2\alpha_2$. When this quantity is negative the sphere shrinks to zero size in the IR which signals that the scalars are massive and that the theory may be in the same universality class as $\mathrm{SU}(N)\times\mathrm{SU}(N)$ Yang-Mills theory. Note that although in this case the form of the metric is the same as (2.4) when the five-sphere shrinks to zero size, we expect the functions α_i to differ since the corresponding gauge groups are different.

3.2.2 (P,Q) D3 branes

Magnetic D3 Branes Consider P magnetic D3-branes placed in a conifold singularity. The discussion will be brief since it is very similar to the electric case. We consider the tachyon potential (3.9) without the quartic term. Using the definition (3.12) in the UV region we find the asymptotic solution

$$\begin{aligned}
T &= -1 - \frac{8}{y} + 100\frac{\log y}{y^2} + \cdots \\
\phi &= \log\left(\frac{2^{14}}{27P}\right) - 2\log y - 25\frac{\log y}{y} + \cdots \\
\xi &= \log(P) - y + \frac{1}{y} + \cdots \\
\lambda &= \frac{1}{2}\log\left(\frac{8}{3}\right) - y + \frac{2}{y} + \cdots \\
\eta &= \frac{1}{2}\log 2 - \frac{y}{2} + \frac{1}{y} + \cdots
\end{aligned} \qquad (3.20)$$

From the form of the dilaton we read the running of the gauge coupling

$$P^{\frac{1}{2}}e^{\frac{1}{2}\phi} \sim \frac{1}{\log u + \frac{25}{8}\log(\log u)}\,. \qquad (3.21)$$

The gravity solution exhibits the UV freedom property and now $b_2/2b_1^2 = 25/8$. The magnetic D3 branes theory is the same as the electric one. However, we see that the gravity description gives different values for b_2 in the two cases. In fact for a general tachyon potential $g(T)$

$$\frac{b_2}{2b_1^2} = \left(\frac{g'(T_0)}{g(T_0)}\right)^2 - \frac{7}{8}\,, \qquad (3.22)$$

where T_0 is the minimum value of the tachyon, which again indicates that the sign of b_2 cannot be captured by gravity.

In the IR limit we find, using $\rho = e^y$ and $y \gg 1$,

$$\begin{aligned}
T &= \frac{16}{y} + 72\frac{\log y}{y^2} + \cdots \\
\phi &= \log\left(\frac{2^{-5/2}}{27P}\right) + 2\log y - 9\frac{\log y}{y} + \cdots \\
\xi &= \frac{1}{2}\log(2P^2) + y + \frac{9}{y} + \cdots \\
\lambda &= \frac{1}{2}\log\frac{8}{3} + y + \frac{2}{y} + \cdots \\
\eta &= \frac{1}{2}\log 2 + \frac{y}{2} + \frac{1}{y} + \cdots
\end{aligned} \tag{3.23}$$

This is precisely the fixed point electric D3 branes solution, except the sign of tachyon. This is due to the sign change of the tachyon tadpole when we go from the electric to the magnetic D3-brane [4].

When considering the tachyon potential (3.9) with the quartic term we can find, as in the electric case, solutions that exhibit confinement in the IR.

Self-Dual D3 Branes Consider now placing N self-dual D3-branes at a conifold singularity. We will use the metric ansatz (3.4) with $h(T)$ given by (3.7). The function $h(T)$ has an extremum at $T_0 = 0$ and the tachyon equation is solved by $T = 0$ with a constant dilaton $\phi = \phi_0$ and

$$\begin{aligned}
\xi &= \log(2N) + \log\rho, \\
\lambda &= \frac{1}{2}\log\left(\frac{8}{3}\right) + \log\rho, \\
\eta &= \frac{1}{2}\log 2 + \frac{1}{2}\log\rho\,.
\end{aligned} \tag{3.24}$$

This solution has the form $AdS_5 \times T^{1,1}$, which indicates that the IR physics of this large N brane system is governed by a fixed point. Note that unlike the self-dual D3-branes in flat space studied in [18], here the theory is asymptotically free. In particular, the self-dual D3-branes theory can be obtained as an orbifold of $\mathcal{N} = 4$ theory by a discrete subgroup of the Z_4 center of the R-symmetry group SU(4) [20]. This is not the case with self-dual D3-branes at a conifold singularity.

4. D3 branes in general backgrounds

What we learned from the previous section is that the supergravity equations for the fields ϕ, T, ξ and η do not depend, at least to the order we checked, on whether we put D3 branes in a flat background or near a conifold. In the following we will

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check this for general backgrounds of the form $M_4 \times Y_6$ where M_4 is flat Minkowski space which corresponds to the D3 brane worldvolume and Y_6 is a non-compact space which is flat or may have a singularity at the origin. The space Y_6 is a cone over a five-dimensional compact manifold X_5 which is called the horizon[1]. We will consider solutions that in the UV and the IR take the form $AdS_5 \times X_5$. For generic values of r the background will be a fibration of X_5 over a five-dimensional space i.e. the metric of the X_5 is multiplied by a radial dependent warp-factor.

We will consider the following ansatz for the metric and the four-form gauge field:

$$\begin{aligned} ds^2 &= e^{\frac{1}{2}\phi} ds_E^2\,, \qquad ds_E^2 = e^{-\frac{1}{2}\xi} dx^\mu dx^\mu + e^{\frac{1}{2}\xi - 5\eta} d\rho^2 + e^{\frac{1}{2}\xi - \eta} d\Omega_X^2 \\ C_{0123}(r) &= A(r)\,, \qquad F_{0123r} = A'(r) \\ \phi &= \phi(r)\,. \end{aligned} \tag{4.1}$$

The effective supergravity action is given by:

$$\begin{aligned} S = \mathrm{Vol}(M_4) \int dr d\Omega_X \sqrt{g} \Big(e^{-2\phi} \Big[-2\Big(R + 4 g^{rr} (\phi')^2\Big) + \frac{1}{2} g^{rr} (T')^2 + \frac{m^2}{2} T^2 \Big) \Big] + \\ + \frac{1}{2} f(T) g^{00} g^{11} g^{22} g^{33} g^{rr} (A')^2 \Big) \\ f(T) = 1 + T + \frac{1}{2} T^2\,, \qquad m^2 = -2\,. \end{aligned} \tag{4.2}$$

The Einstein equations arising from variations with respect to the metric components with X_5 indices give one part which is proportional to the Ricci tensor plus terms that are proportional to the X_5 metric. So in general the metric of the horizon has to obey the following equation

$$R_{ij} = c g_{ij}\,, \tag{4.3}$$

where i, j are X_5 indices. It is easy to see that we can make the choice $c = 4$ or in other words X_5 can always be chosen to be an Einstein space. Note that a different choice of the constant c can be absorbed by constant shifts in ξ, η and ϕ. Well known examples of Einstein spaces are S^5 in the case of a flat background, the homogeneous spaces $T^{p,q}$ and del Pezzo surfaces.

In all these cases the equations will be of the form (3.10) with $\lambda = 2\eta$ (up to a constant) and asymptotic solutions will take the form (3.13), (3.16) up to possible constant shifts in ξ, η and ϕ.

[1] This is similar to what happens in type IIB theory with the same background. The supergravity equations yield the same harmonic function for the D3 branes $f = 1 + Q/r^4$ as in a flat background. In the type 0 the functions ξ and η take over the role of the harmonic function f and do not depend on the geometry of the horizon as well.

This means that in general backgrounds of the form $M_4 \times Y_6$ we will find solutions that correspond to asymptotically free theories in the UV, and there will always exist a solution in the IR which describes a non-trivial conformal fixed point at infinite coupling. We can also construct in the IR a large class of confining theories with massgap along the lines of section 3.2. One important issue is whether there is a gravity description of the flow from the UV to the IR. This will not be in general the case, as we will discuss in the next section.

Note that in view of the above discussion the two-loop beta function determined by the gravity description is independent of the gauge theory we have on the branes and is universal.

5. Discussion

Besides the Type 0 examples, there are by now several gravity solutions discussed in the literature that may provide a dual description of confining non supersymmetric gauge theories [11, 12, 21, 22, 23, 24]. In all of them the gravity solution cannot provide a quantitative description.

In the following we will discuss the issue of finding solutions in the effective Type 0 gravity theory that describe qualitatively the whole RG flow from the UV to the IR. To be specific we discuss the case of electric branes in flat space, but in view of the previous section the discussion is similar for branes at singularities. On the UV side we found a good candidate solution that exhibits asymptotic freedom which has a description in terms of classical string theory. Using numerical methods we tried to continue the known asymptotic solution in the UV to the IR. In the case of zero quartic coupling $t = 0$ we found that the tachyon goes from $T_0 = -1$ through an oscillatory region and settles down at zero. Unfortunately the numerical approximation fails at that point and one is not able to trust the numerical method and go beyond.

The other possibility is to start with one of the IR solutions (the IR fixed point or the confining solutions) and proceed to the UV. Whereas starting from the IR fixed point one runs into similar numerical problems as for the UV solution, starting with a confining solution in the IR, which is the most generic situation in field theory, produced the most promising results. In this case we chose $t = 1/4$ such that $g'(T_0) = 0$ and the tachyon does not run and is constant $T_0 = -1$ all along the flow. In this situation we were able to find solutions connecting the IR to the UV but these solutions depend strongly on the integration constants $\alpha_i, \phi_0, \xi_0, \eta_0$. In some cases ϕ, ξ, η settle down at constant values in the UV which corresponds to a space time with geometry $\mathbb{R}^+ \times \mathbb{R}^4 \times S^5$. The field theory interpretation of such a solution in the UV is missing. In other cases one or several of the functions diverge, but we were not able to identify a solution in the UV that asymptotes $AdS_5 \times S^5$ and behaves like the UV solution. Still, the good behaviour of the solutions we found gives us some confidence that such a solution exists, but the integration constants have to be chosen carefully. A generic confining solution in the IR in general does not connect to a UV solution with asymptotic freedom. To further pursue this direction requires a better understanding of the relation between the field theory parameters and the integration constants.

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